

A STUDY ON FATIGUE CRACK GROWTH IN THE PRESENCE OF A HOLE IN THE VICINITY OF THE CRACK PATH.

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

**BACHELOR OF TECHNOLOGY
IN
MECHANICAL ENGINEERING (2011-15)**

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**NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA**

CERTIFICATE

This is to certify that the project entitled, “**A STUDY ON FATIGUE CRACK GROWTH IN THE PRESENCE OF A HOLE IN THE VICINITY OF THE CRACK PATH**” submitted by **Ashish Chandrakar (111ME0286) and Ankit Agrawal (111ME0281)** in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in **Mechanical Engineering** at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance. To the best of our knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

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ACKNOWLEDGEMENT

Successful completion of work will never be one man's task. It requires hard work in right direction. We wish to express our deep sense of regard and extreme gratefulness to **Prof. P.K. Ray, Department of Mechanical Engineering, N.I.T Rourkela**, for introducing this project and for his inspirational guidance, constructive ideas and valuable suggestion throughout our project work.

We also extend our sincere thanks to **Mr. Ajith Kumar** and **Mr. Vaneshwar Kumar Sahu** for their constant support during the project work. We would also like to thank Cornell Fracture Group, Cornell University, for making the software **CASCA** and **FRANC2D** available free of cost.

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ABSTRACT

The presence of a hole in the vicinity of a growing fatigue crack may lead to deviation of the crack path. If the hole is too near the crack path, the crack may terminate at the edge of the hole, leading to crack arrest. In this project, a finite element based two dimensional crack propagation simulator software FRANC2D and a pre-processor software for this simulator CASCA developed by Cornell Fracture Group of Cornell University was used for prediction of crack propagation in a beam containing a hole. Four point bending test experiment was carried out on an aluminum beam and crack growth propagation behavior was observed. These two observations i.e. from FRANC2D and experiment were compared.

KEYWORDS: FRANC2D, Four Point Bend Test, hole-crack interaction

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1. INTRODUCTION

1.1 Background

Fatigue is termed as the course of progressive localized permanent structural change taking place in a material exposed to conditions that produce fluctuating stresses at some point or points and that may end in cracks or complete fracture after a sufficient number of fluctuations. Actually it is behavior of materials under cyclic loading. The stress value in case of fatigue failure is lower than ultimate tensile stress and may be even lower than the yield stress limit of the material. Generally, fatigue loading indicates cyclic variation of stress and strain in a component. Often machine members subjected to repeated or cyclic stressing are found to fail when the actual maximum stresses are below the ultimate strength of the material, and quite frequently at stress values even below the yield strength. Fatigue is estimated to cause 90% of all failures of metallic structures or components such as bridges, aircraft, machine components, etc. are occurring under fluctuating / cyclic stresses, failure can occur at loads considerably lower than tensile or yield strengths of material under a static load. [1]

Fatigue failure begins with a small crack; the initial crack may be so minute and cannot be detected. The crack usually develops at a point of localized stress concentration like discontinuity in the material, such as a change in cross section, a keyway or a hole. Once a crack is initiated, the stress concentration effect become greater and the crack propagates. Consequently the stressed area decreases in size, the stress increase in magnitude and the crack propagates more rapidly. Until finally, the remaining area is unable to sustain the load and the component fails suddenly. Thus fatigue loading results in sudden, unwarned failure.

Most of the mechanical components experience fluctuating load due to change in:

- Load Magnitude
- Load Direction
- Load application point

1.2 Four point bend test

In four point bend test beam specimen is restricted at four points, two farther points act as support and two points nearer to center act as load bearing points. Hence, there is pure bending in mid span of beam. The 4 point bend test produces peak stresses along a prolonged region of the specimen hence revealing a larger length of the specimen with more potential for defects and flaws to be emphasized whereas the 3 point flexure fixture produces its peak stress at the specimen mid-point with reduced stress elsewhere. The stress localization is ideal for testing for specific isolation of stress on a component or material. [2]

2. LITERATURE REVIEW

In materials science, fatigue is the weakening of a material caused by recurrently applied loads. It is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. The nominal maximum stress values that cause such damage may be much less than the strength of the material typically quoted as the ultimate tensile stress limit, or the yield stress limit.

ASTM defines fatigue life, N_f , as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs. For some materials, notably steel and titanium, there is a theoretical value for stress amplitude below which the material will not fail for any number of cycles, called a fatigue limit, endurance limit, or fatigue strength. Engineers have used any of three methods to determine the fatigue life of a material: the stress-life method, the strain-life method, and the linear-elastic fracture mechanics method. [3]

Several studies have been carried out till date in order to analyse crack hole interaction. It has been shown, for bi-dimensional cases, that the X-FEM analysis provides an interesting explanation of observed experimental dynamic crack propagation. In the case of the interaction of a crack path with a hole, it has been first observed that the crack avoids the hole with the dynamic loading and falls into the hole if loaded by a quasi-static similar one.[4]Murakami et al. [5], [6], [7] and [8][9] have carried out over the last 30 years an extensive experimental research program in the field of fatigue crack growth from material inclusions or artificially introduced defects. A reliable estimation of the fatigue strength needs a reliable estimation of the maximum defects occurring in a piece. This can be done by applying the statistics of extremes to the analysis of defects [10]. Defects such as pores or non-metallic inclusions are a prominent cause of fracture for high strength components subject to high cycle fatigue (HCF). These defects cause stress concentrations in the material and are thus preferential sites for crack nucleation. The result is that one sufficiently large defect is enough to cause fracture in a component. Nucleation of a fatigue crack from a defect depends very strongly on its size and shape, the strength of the matrix material and adhesion between the defect and the metal matrix. Defect size has experimentally been found to be significant. Murakami [11] has made extensive studies of the long life fatigue strength of materials containing small cracks, holes, inclusions, porosity and other inhomogeneities. He shows that the

fatigue limit is not the critical stress for crack nucleation but the threshold stress for propagating a crack which emanates from the original defect. Once a crack is initiated, the stress field around the crack tip dominates crack extension into subsequent grains. [12]

The stresses near a crack tip can be characterized by a lone factor called the Stress Intensity Factor (SIF). As the stress intensity factor can be used for the quantitative estimation of fracture strength of the defective structure. The engineering importance of fracture mechanics applications depend upon the stress intensity factors. Till today, there have been over 20 approaches to calculate stress intensity factors. Some of these are the integral transform method [13], the complex variable function method [14], the singular equation integral method [15], conformal mapping [16], the Laurent series expansion [17], boundary collocation method [18], Green's function method [19], the continuous distribution dislocation method [20], the finite element method [21], the boundary element method [22], the body force method [23] and the displacement discontinuity method [24], the Westergaard method [25], etc.

The historical development of computational fracture mechanics is found in the works of Ingraffea and Wawrzynek [26] and Sinclair [27]. Paul Wawrzynek and Anthony Ingraffea developed FRANC-2D. The solutions of many of the fracture mechanics problems have been compiled in [28, 29] for stress intensity factors. Sinclair [26] has presented an extensive review of stress intensity factor numerical prediction models. The advantages and disadvantages of using finite element in computational fracture mechanics have been well addressed by Ingraffea [30]. Miranda et al. [31] have discussed on the aspect of mesh refinement and associated error in computing stress intensity factors using finite element method. It has been noted that too much mesh refinement may significantly lower the calculation accuracy in crack problems.

3. EXPERIMENT DETAILS

The tests were carried out in a dynamic universal testing machine. The details of the machine are given below: -

Machine Specification:

Universal Testing Machine

Company: BISS (Bangalore Integrated System Solution)

Max Load: 100 KN

Specimen Specification:

Aluminum Beam

Mechanical Properties:

Ultimate tensile strength	111 MPa
Yield tensile strength	68.9 MPa
Modulus of Elasticity	68.5 GPa
Poissons Ratio	0.33

Table 3.1: Mechanical Properties of the material of the beam

Composition:

Component	Percentage composition
Aluminum	99.5
Iron	0.284
Zinc	0.0572
Copper	0.0561
Magnesium	0.0518
Vanadium	0.0469

Table 3.2: Composition of the material of the beam

Details of Beam specimen

A $25 \times 25 \text{ mm}^2$ cross section single edge notched beam made of aluminum material was used for our experiment. The beam specimen had initial planar notch at one plane of having length 2.951 mm. The notch was machined by wire EDM machining process. The notch was straight and at the middle along its length. The details of the beam specimen are shown in fig 3.1.

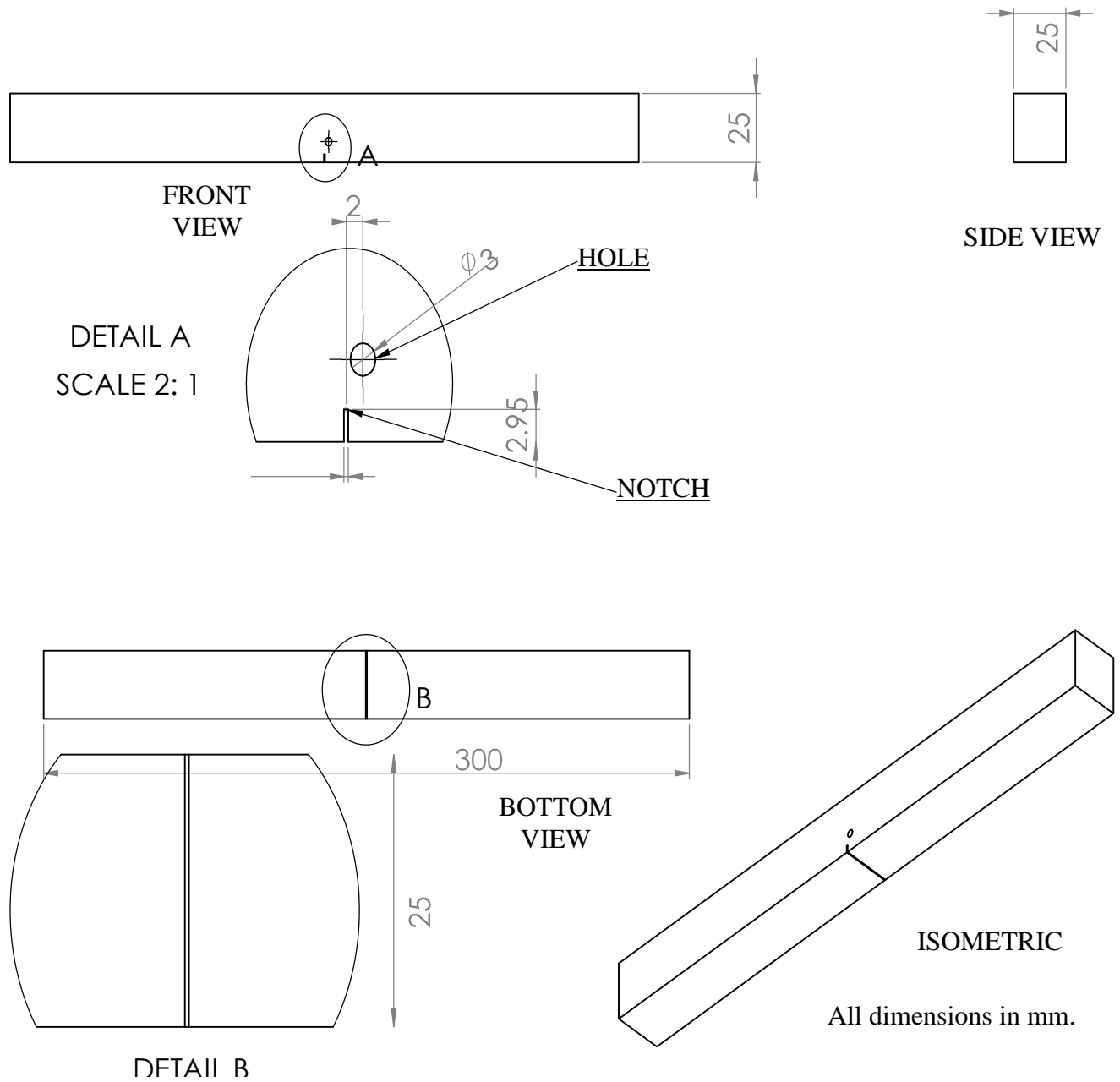


Fig 3.1: Beam Specimen [32]

Specification for four point bend test:

Load (p_{\max}): 7.8 KN

Loading frequency: 5 Hz

Stress Ratio ($R = \frac{P_{\min}}{P_{\max}}$): 0.3



Fig 3.2: Loaded Specimen

4. FRANC-2D

The Fracture analysis Code (FRANC2D) was originally developed by Paul Wawrzynek at Cornell University. FRANC2D represented a significant step in the development of discrete fracture analysis programs because of its modular software design and topological data structure.[33] FRANC2D models can be created with CASCA, which is a simple pre-processor. Franc-2D analysis is done in two parts first geometry and mesh have to be defined in CASCA.

4.1 Geometry and initial mesh generation-CASCA

CASCA is a pre-processor for creating input files for FRANC-2D. It is used to generate initial meshes and geometry. An initial mesh has to be performed before a franc-2D simulation can be performed.

Building initial mesh with CASCA:

- Begin by running CASCA program. Initially there were three types of options set scale, read and adjusting your view with reset, magnify, pan, zoom and snap.
- **Creating problem outline.**

A basic 2D geometry was created by using ‘get line’ and ‘get circle’ options. Then the hole was specified using ‘specify hole’ and selecting the region inside circle. Final geometry of beam is shown below.

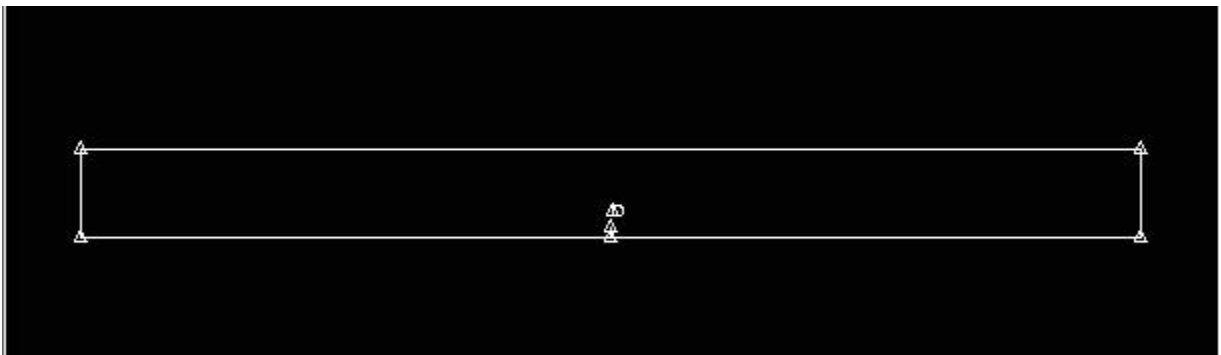


Fig 4.1: Basic 2D Geometry of Beam specimen in CASCA

- **Adding sub regions.** This option helped to break object into number of simpler regions that are more convenient for meshing. We divided our beam into 10 regions as shown. Regions were subdivided using get line option.

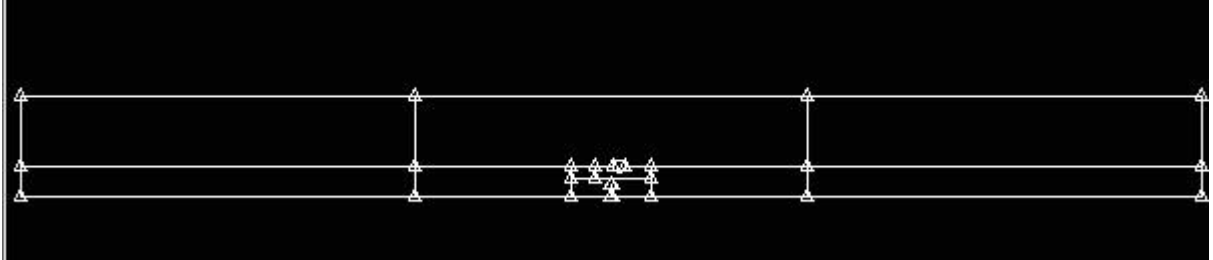


Fig 4.2: Subregions

- **Adding subdivision.** Subdivision was used to specify nodal densities along boundaries for all regions in structure. Graded nodal density was specified for finer mesh quality.

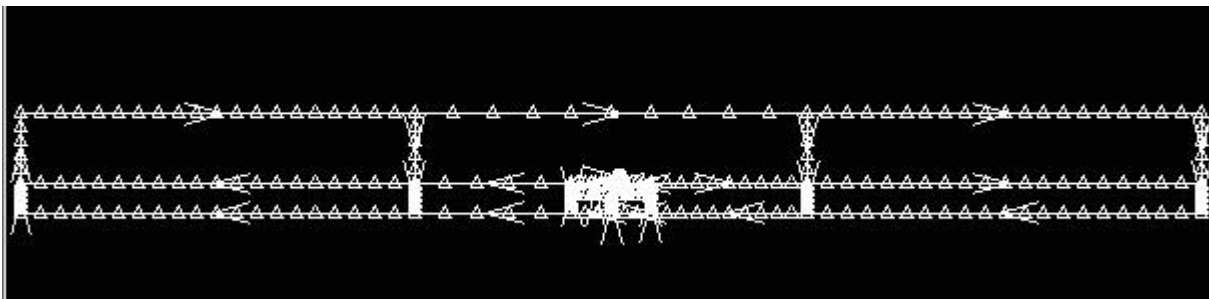


Fig 4.3: Subdivision

- **Mesh generation.** Bilinear 4 sided meshing algorithm was used to mesh this structure. This algorithm requires a rectangular region with equal number of nodes on opposing sides.

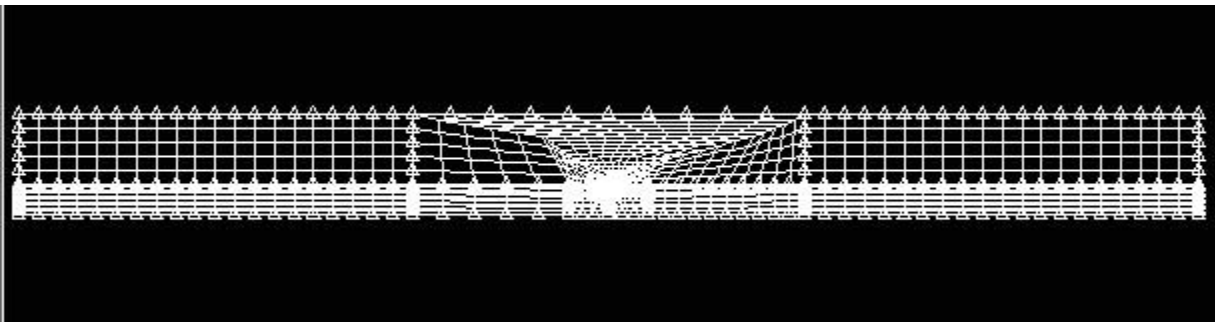


Fig 4.4: Mesh Generation

4.2 Crack propagation in FRANC2D

The steps for crack initiation and propagation in FRANC2D are [34] [35]:

- After the geometrical design and meshing of the beam CASCA, the mesh file was saved in .inp format.
- The mesh file saved in .inp format was opened in FRANC2D.
- Then the problem type was specified to plain stress condition and appropriate material properties were given for the beam.
- Material of beam was specified by selecting MATERIAL command. Young's modulus, Poisson's ratio and thickness values were given by selecting E, NU and THICKNESS options respectively. Then the element stiffness Matrices was reformulated which was done by selecting ELEM STIFF option, and then the file was saved.
- Boundary conditions specification: This was done by selecting PRE- PROCESS and then FIXITY option. Two nodes or ends were fixed appropriately in XY direction .The size of the box containing the node was adjusted using the tolerance window given at the left hand below corner.
- Loads were given by selecting LOADS -> POINT LOAD. Then the corresponding values of load were entered at specified location of the beam.
- Before crack initiation stress analysis is must which was done by selecting ANALYSIS -> LINEAR -> DIRECT STIFF.
- After the analysis was done, to see whether boundary conditions were properly given or not we selected DEFORMED MESH option. Then POST-PROCESS option was selected, followed by CONTOUR option to view various color stress contours which indicate principle tensile stress(SIG 1) , effective stress(EFF STRESS), shear stress(TAU MAX) etc..

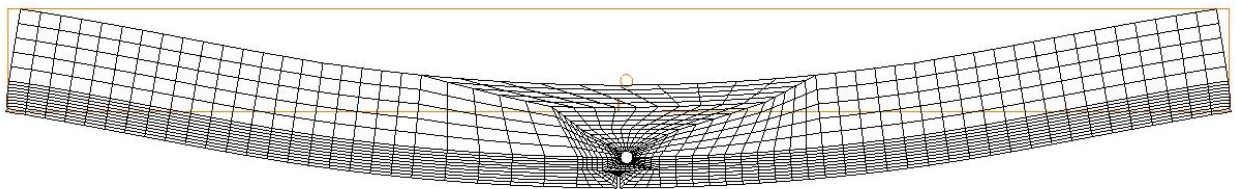


Fig 4.5: Deformed Mesh Structure

- Now an initial crack was initiated in the beam which was done by selecting MODIFY -> NEW CRACK ->TRACTION FREE-> EDGE CRACK.
- The location of the notch was at the middle of the beam. The minimum number of elements along crack extension was taken as 3. Then the ACCEPT option was selected and Re-meshing of nodes took place.
- For this new structure, new analysis was performed by selecting ANALYSIS -> LINEAR -> DIRECT STIFF.
- Now the crack was propagated along the width from the crack tip. This was done by entering MODIFY -> MOVE CRACK -> AUTOMATIC -> PROPAGATE. To give the specified amount of crack growth at each step CRACK INCR option was chosen and crack increment value per step was specified. STEPS option was then used to set the no. of propagation steps at each propagation. Then PROPAGATE option was selected to begin crack propagation.
- Then the file was saved using WRITE option.
- Now the fatigue crack growth analysis was done by selecting POST -PROCESS and FRACT MECH options. The stress intensity factor history was found using SIF HISTORY option. A KI vs. crack length graph was generated. Here KI is stress intensity factor.

Figures shown below [figs. 4.6-4.9] represent fatigue crack propagation in Franc 2D under different cases of hole in SENB specimen.

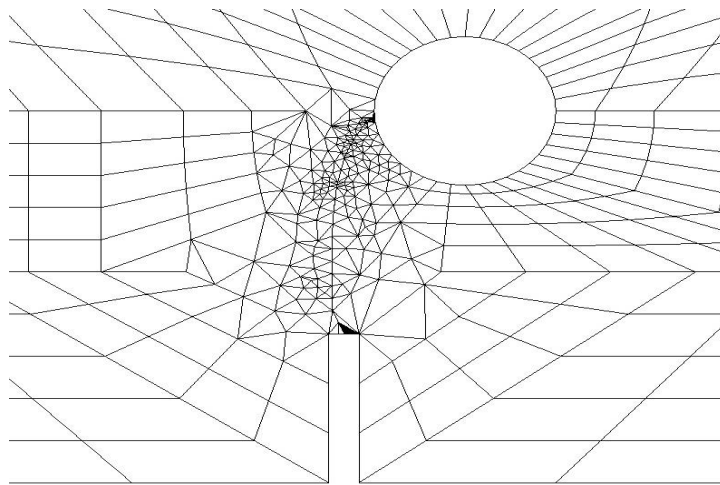


Fig 4.6: Franc 2D Crack Propagation, Center to Hole Distance (x-axis): 2mm

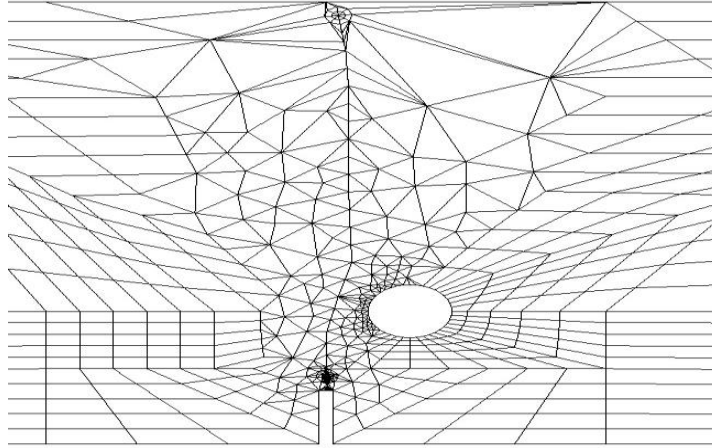


Fig 4.7: Franc 2D Crack Propagation, Center to Hole Distance (x-axis): 3mm

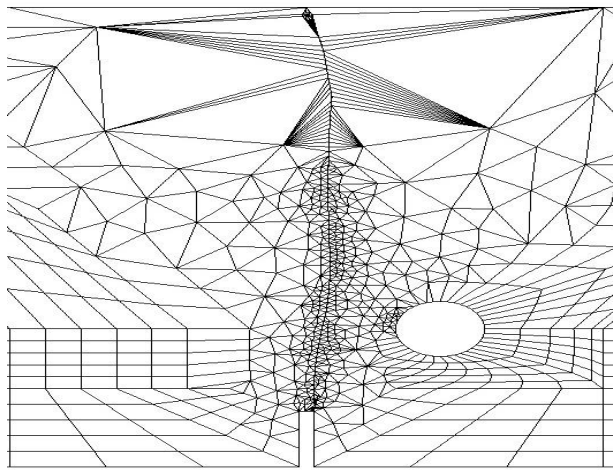


Fig 4.8: Franc 2D Crack Propagation, Center to Hole Distance (x-axis): 4.5mm

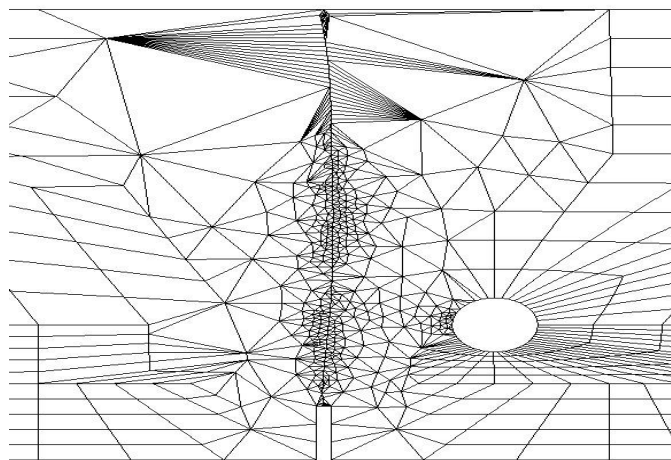


Fig 4.9: Franc 2D Crack Propagation, Center to Hole Distance (x-axis): 6mm

From the above figures, it was found that when hole is at a distance of 6mm from center the crack propagates almost linearly in y direction and when this center distance is decreased the crack was found to be deviating towards it and finally in the case when hole is at a distance of 2mm is crack is shown to merge with the hole.

5. DATA ANALYSIS

5.1 EXPERIMENTAL DATA ANALYSIS

From the four point bend test performed on the beam, various data were recorded vis-à-vis ‘x’ and ‘y’ coordinate and number of cycles. Then the crack length was calculated using x and y coordinates and a graph was plotted between crack length and no. of cycles. Detailed information obtained during the fatigue crack propagation is tabulated below.

X axis	Y axis	a	n
5.758	6.26	0	0
5.736	6.24	0.297321	25350
5.73	6.175	0.89493	26053
5.728	6.157	1.0728	26703
5.716	6.12	1.461643	28104
5.759	6.091	1.69003	29403
5.775	6.039	2.216529	30201
5.746	6.045	2.153346	31624
5.736	6.008	2.529585	32800
5.736	5.955	3.057924	34006
5.85	5.91	3.618895	35466

Table 5.1: Crack Propagation Information

5.1.1 No. of cycles vs Crack length

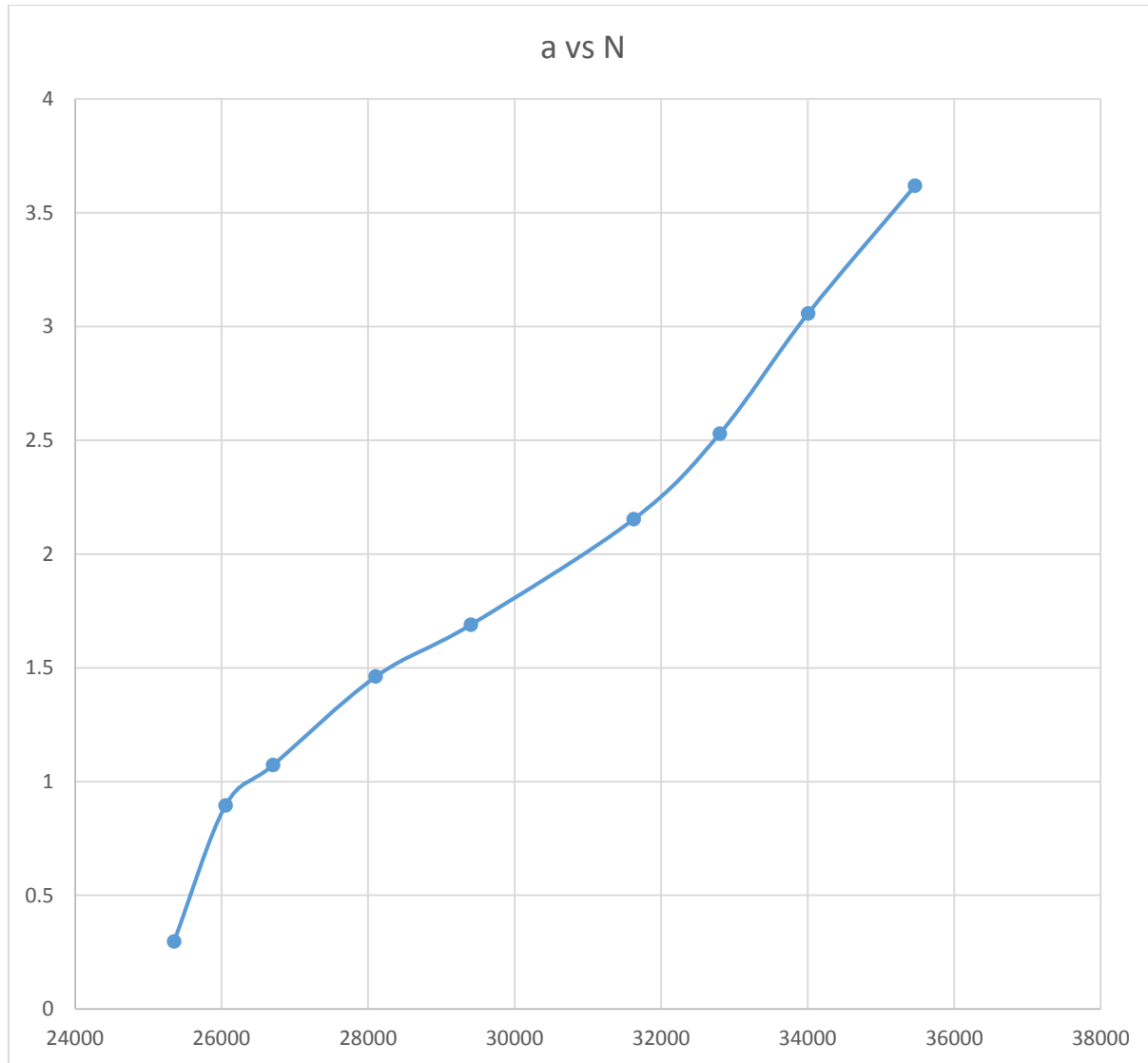


Fig 5.1: Crack Length vs. No. of cycles

5.1.2 a_y vs N and a_x vs N

For refining crack path data experimental values of x and y coordinates of crack path were plotted against N independently, and curve fitting of the plotted data was carried out using MATLAB.

X axis	Y axis	N
0	0	24000
-0.22	0.2	25350
-0.28	0.65	26053
-0.3	1.03	26703
-0.42	1.4	28104
0.01	1.69	29403
-0.12	2.15	30201
0.17	2.21	31624
-0.22	2.52	32800
-0.22	3.05	34006
0.92	3.5	35466

Table 5.2: Crack path coordinates

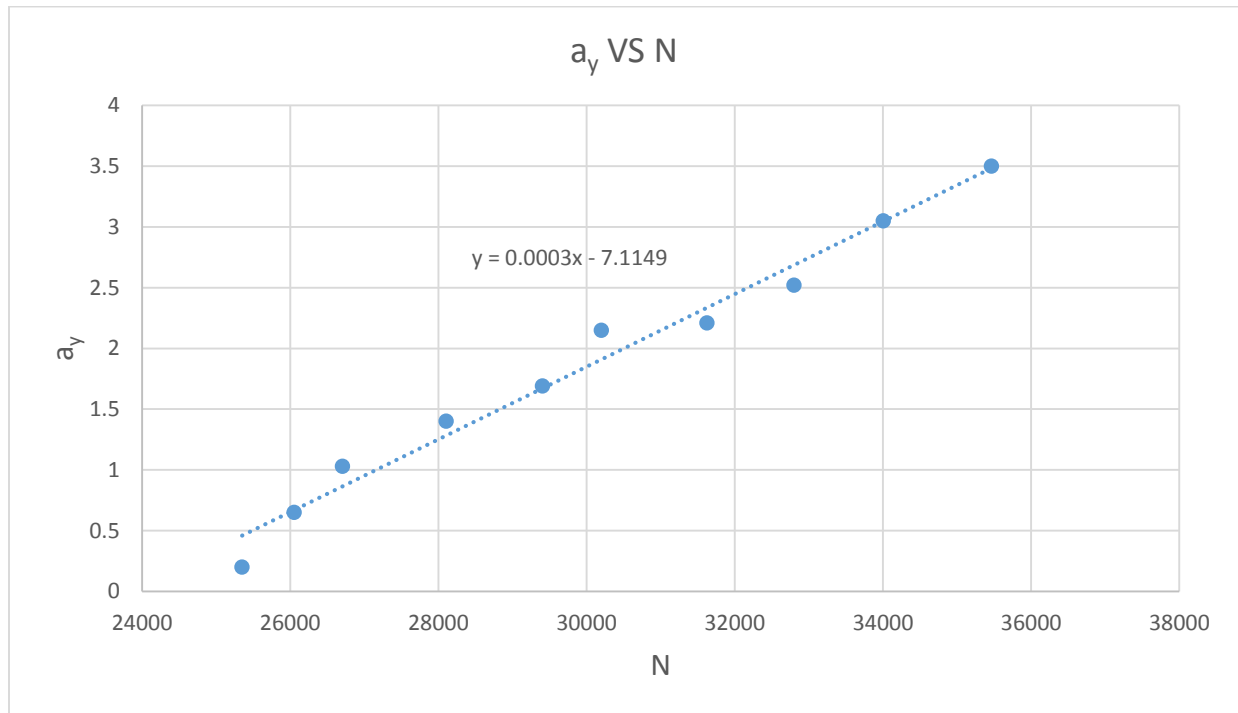


Fig 5.2: a_y vs N

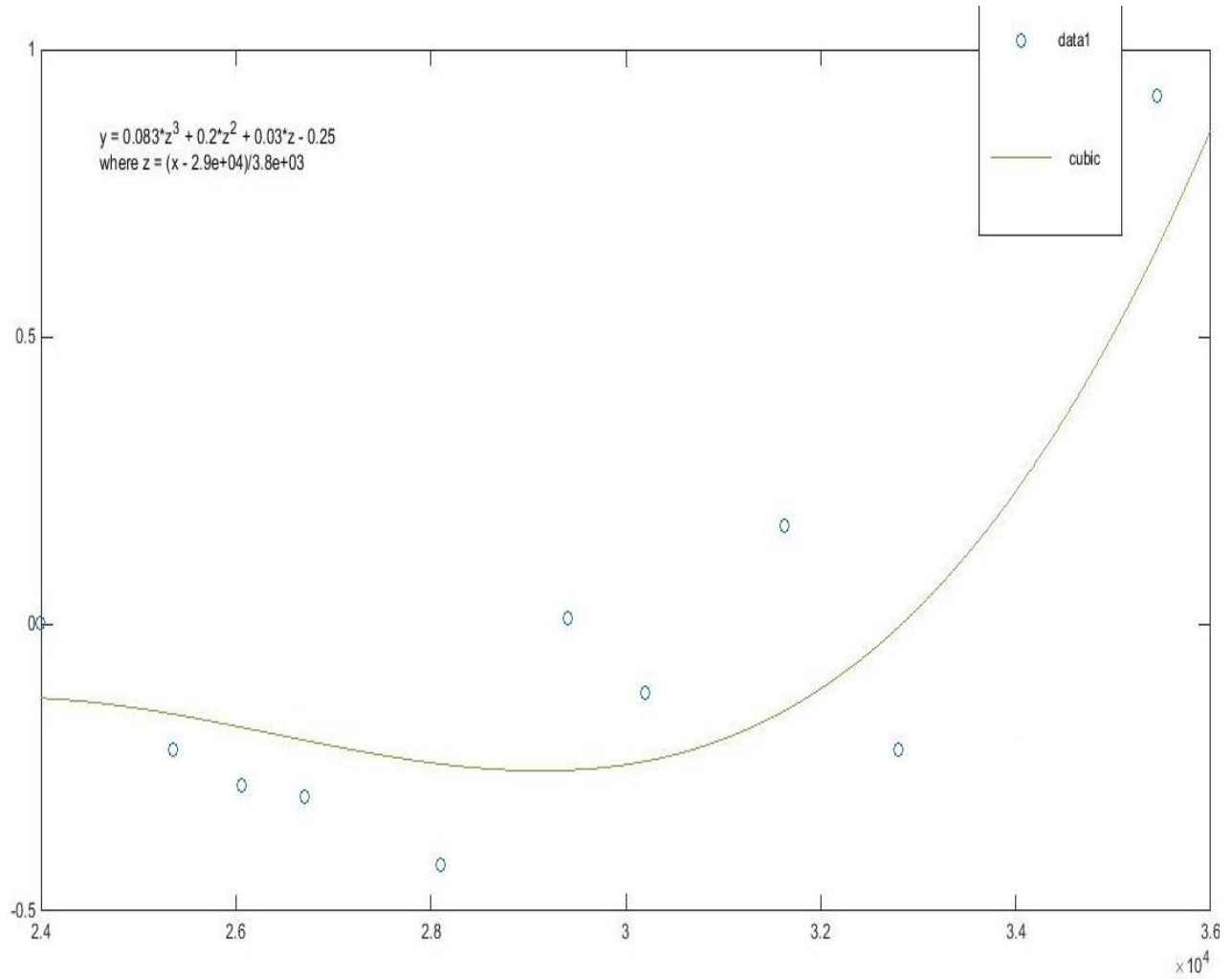


Fig 5.3: a_x vs N

The equations obtained through Matlab Curve fitting [36] are shown in the figs. 5.2 and 5.3 are:

$$a_y = 0.0003x - 7.1149$$

$$a_x = 0.083z^3 + 0.2z^2 + 0.03z - 0.25$$

$$\text{where, } z = (x - 2.9e+04)/3.8e+03$$

From this equation new value of x and y coordinates were calculated as shown in table 5.3.

N	z(center and scale data)	X	Y
25000	-1.052631579	-0.15678	0.3851
26000	-0.789473684	-0.18987	0.6851
27000	-0.526315789	-0.22249	0.9851
28000	-0.263157895	-0.24556	1.2851
29000	0	-0.25	1.5851
30000	0.263157895	-0.22674	1.8851
31000	0.526315789	-0.16671	2.1851
32000	0.789473684	-0.06082	2.4851
33000	1.052631579	0.099993	2.7851
34000	1.315789474	0.32481	3.0851
35000	1.578947368	0.622707	3.3851
36000	1.842105263	1.002759	3.6851
35466	1.701578947	0.789038	3.5249
37000	2.105263158	1.474041	3.9851
38000	2.368421053	2.04563	4.2851
39000	2.631578947	2.7266	4.5851

Table 5.3: Curve fitting through equations obtained from Matlab

5.2 Franc 2D Data Analysis

Crack path coordinates and number of cycles(N) vs crack length(a) obtained from FRANC-2D are shown in table 5.5 and fig 5.7.

x- franc2d	y-franc2d
0	0
-0.089	0.237
-0.184	0.47
-0.198	1.22
-0.194	1.97
-0.168	2.47
-0.119	2.97
-0.024	3.46
0.133	3.93
0.229	4.1
0.325	4.26
0.391	4.36
0.436	4.4
0.49	4.44

Table 5.4: Crack Path coordinates of Franc 2D

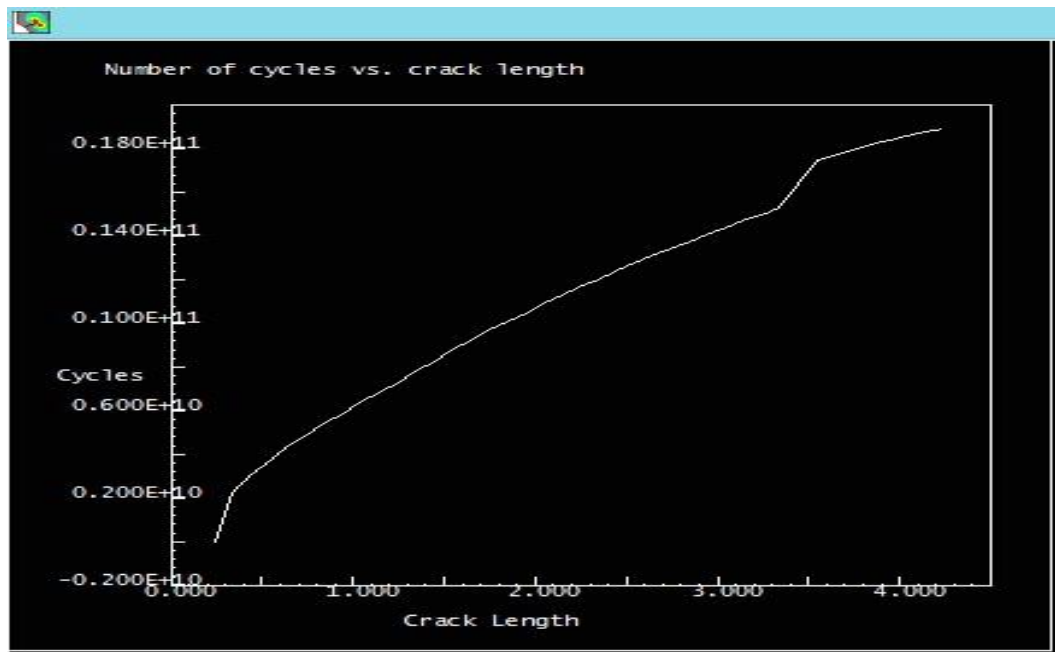


Fig 5.4: Number of cycles vs Crack length (Franc 2D)

5.3 Comparison of crack propagation

Crack propagation paths were compared for the below three cases:

- Experimental data
- Franc-2D
- Refined experimental data

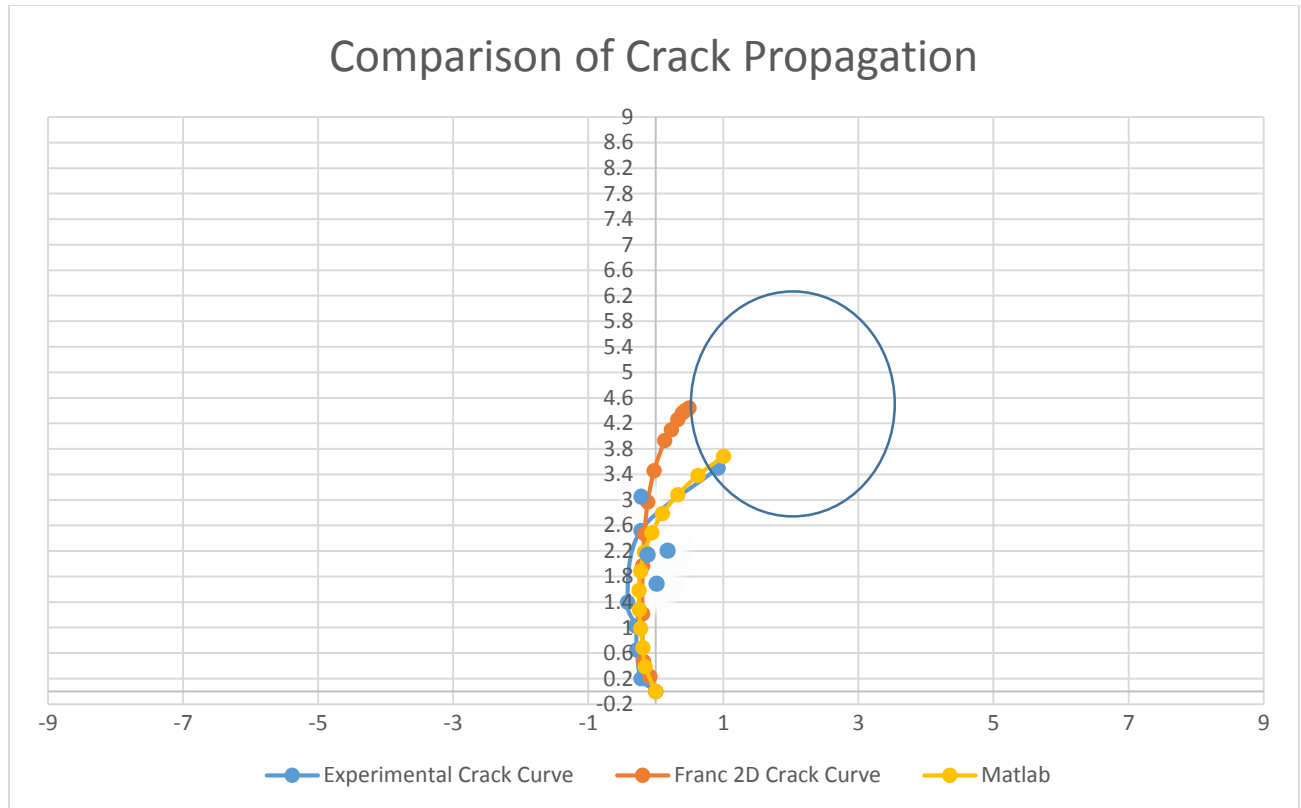


Fig 5.5: Comparison of Crack Propagation

6. CONCLUSION

In the present work, the study of fatigue crack growth in vicinity of a hole was carried out by conducting four point bend test on SENB (Single Edge Notched Beam) specimen with a hole. Crack propagation path from Franc-2d, experimental data and refined experimental data were plotted and it was observed that crack path overlaps up to a certain point and thereafter deviation from simulated path was detected. In fatigue experiments there is a lot of scatter of data. It is therefore necessary to conduct a large number of experiments with large number of specimens and work out a mean path. This will possibly reduce the amount of deviation from simulated crack path. Further it can be concluded that fatigue crack path gets deflected in presence of hole and even merges with the hole if the offset distance is small enough.

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